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Modelling epidemics with diffusion

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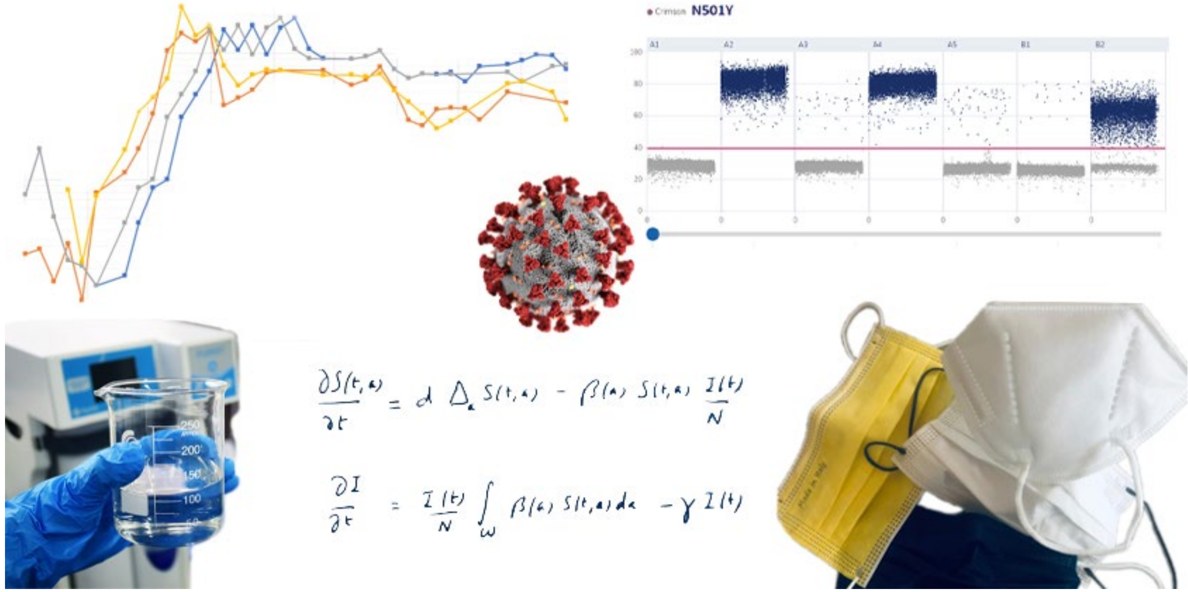
Modelling the Propagation of Epidemics with Diffusion

Henri Berestycki
(EHESS, Paris, and Univ. of Maryland, College Park

- Tamkang Clement and Carrie Chair
- 19 March 2024

I. INTRODUCTION



Epidemics have always existed in human history

Epidemics: diseases transmitted by contagion →

Invasion of populations causing deaths or illnesses

Then diminish in severity, or disappear, or become endemic

Recurrences

Some historic pandemics

"Spanish flu" in 1918-19 caused over 50 million deaths worldwide

"Black death" (plague) in the 14^{th} century, arrived in Europe from Asia in 1346, wiped 1/3 of population in Europe between 1346 and 1350

Japanese smallpox, 735-737, killed 1/3 of population in Japan

Many endemic diseases with large mortality: tuberculosis, measles, malaria, HIV/ AIDS

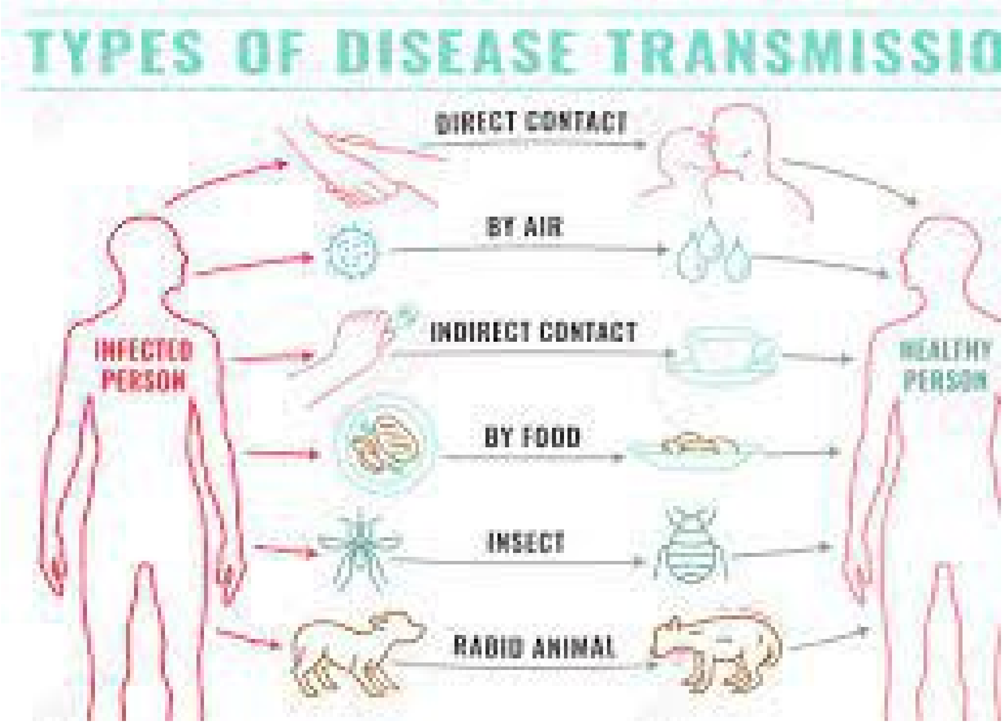
Pandemics: major concern of public health

In the 21st century:

- SARS-CoV (2002-2003)
- H1N1 (2009)
- Ebola (1976, 2013)
- MERS-CoV (2012)
- HIV/AIDS (estimated to have killed 43 million worldwide)
- **SARS-CoV-2 or COVID-19, Dec. 2019 (estimated between 7 and 35 million deaths worldwide)**
- …

Epidemics: *Communicable diseases*

- Epidemics: Direct transmission through individual-to-individual contact
- Contagion mechanism: physical contacts, by particles in the air, insects, animals…

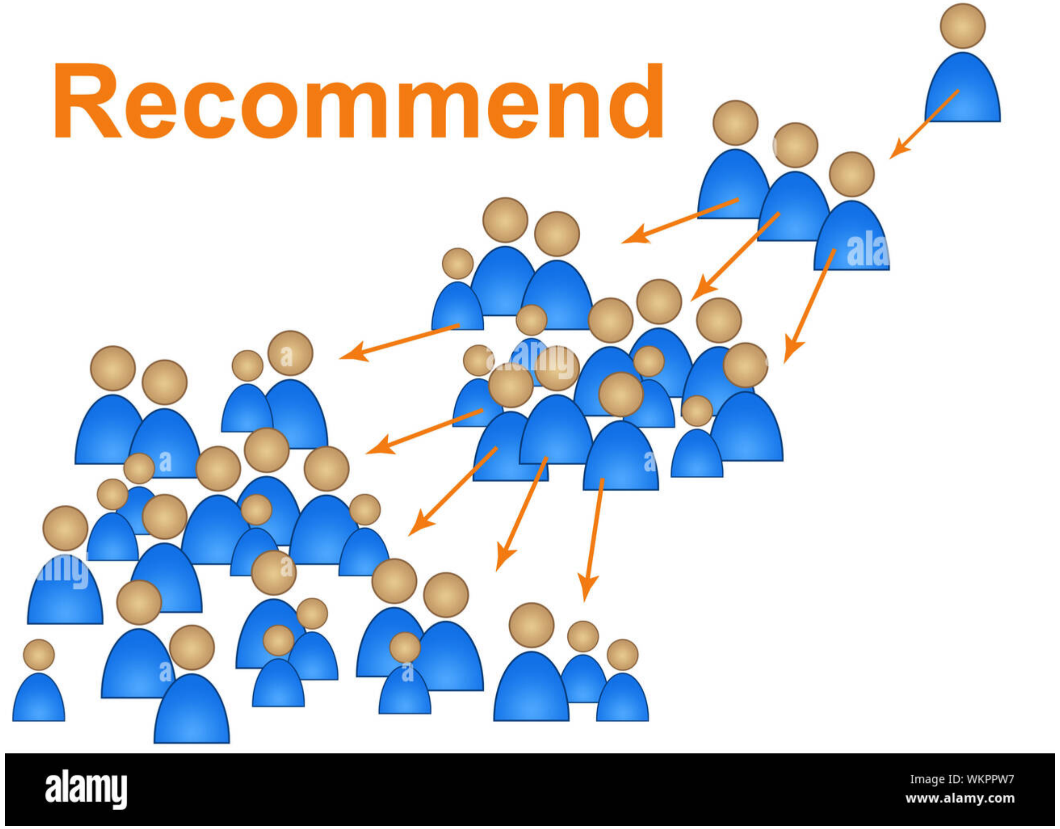


Mathematical modelling of epidemics

- Mathematical laws describing dynamics of an epidemics
- Understanding spreading
- Forecasting
- Devising ways to control it

Many other phenomena with contagion

- Dynamics of opinions
- Social norms
- Spreading on social networks
- Marketing: the Bass model and variants
- Adoptions of new technologies…

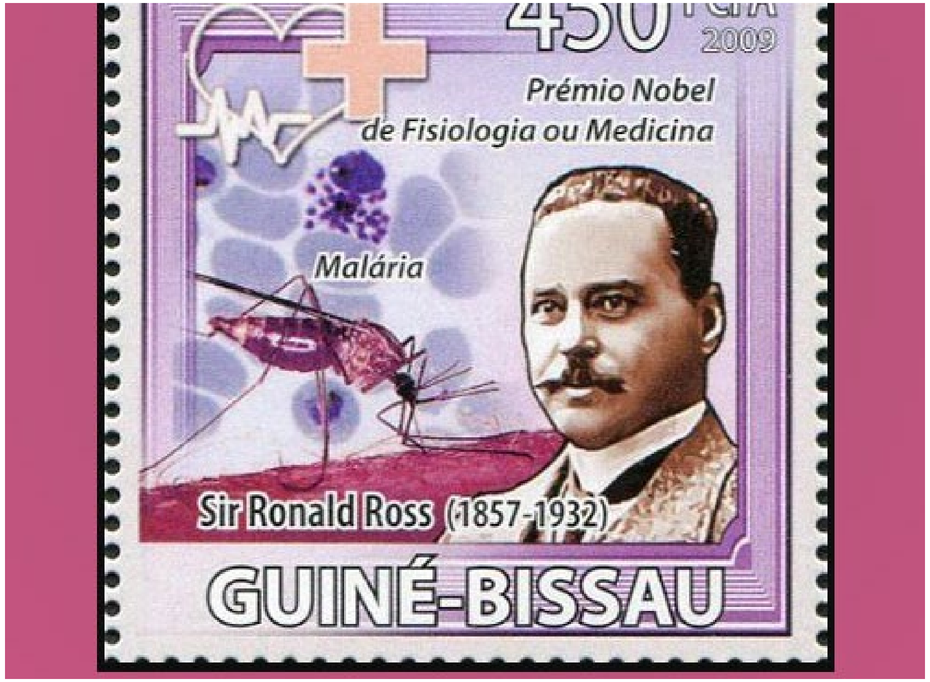


Compartmental Models in Epidemiology

- Introduction of compartmental models, beginning of 20 th century: Ross, Hammer…
- Large, homogeneous population (notion of density)
- Population divided into **compartments** :
  - Susceptibles → Infected → Removed (recovered)
  - Exposed, Asymptomatic, Hospitalized,…
- Describe flow of populations from one compartment to another

Ronald Ross (1857-1932)

- MD, worked in the Indian Medical Service
- Discoverer of malaria transmission
- Second Nobel Prize in Physiology or Medicine (1902)
- A polymath:
  - Poet
  - Novelist, Playwriter
  - Artist
  - Musician
  - Mathematician



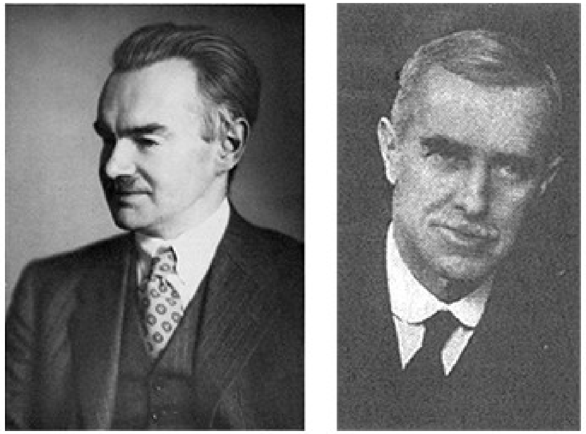
Ronald Ross and Hilda Hudson

- *An application of the theory of probabilities to the study of a priori pathometry, part I* (1916)
- II and III with Hilda Hudson (1917)
- Papers in geometry

II. The SIR Model

Ross, Hammer, Kermack and McKendrick

(all working in public health medical services)



W. Kermack (1898-1970), A.G. McKendrick (1976-1943)

Kermack and McKendrick Model of Epidemiology (1927)

- Large, homogeneous population (density)
- 3 **compartments** :

Susceptibles → Infected → Removed

- I contaminates S until removed
  → R
- Removal and contamination rates depend on duration of infection
- Contamination depends on transmission rate

The SIR system

- A system of Ordinary Differential Equations (ODEs)

$$
\begin{cases}
\dfrac{dS}{dt} = -\beta S I, \\[2ex]
\dfrac{dI}{dt} = \beta S I - \gamma I, \\[2ex]
\dfrac{dR}{dt} = \gamma I
\end{cases}
$$

β = Transmission rate (depends on number of contacts, transmissibility, social distancing etc…

$\beta = \tau\chi$ = product of rate of transmission upon contact by rate of social contact

$\frac{1}{\gamma}$ = Duration of infection

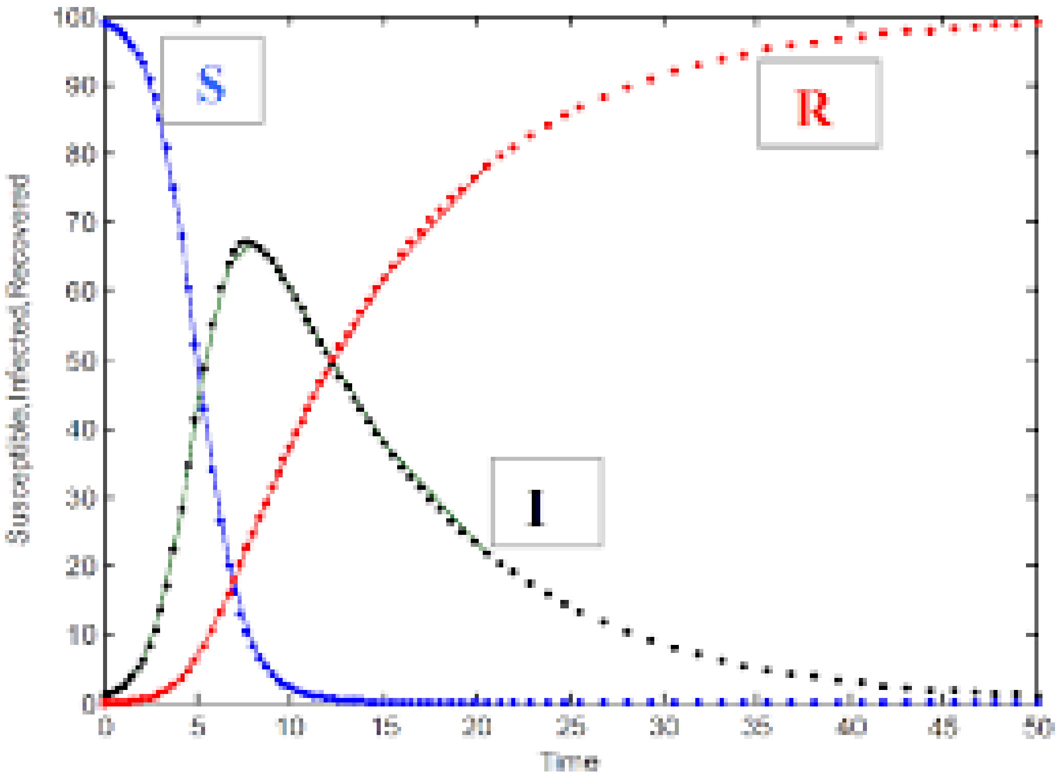
$R_0 = \frac{\beta S_0}{\gamma}$ = Basic reproduction number, to be compared with 1

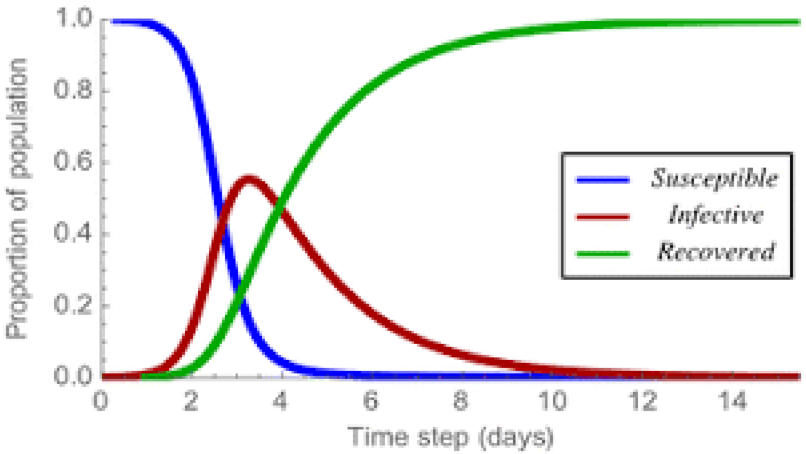
The parameters of the SIR model

Classical SIR model

$$
\begin{cases}
\frac{dS}{dt} = -\beta S I \\
\frac{dI}{dt} = \beta S I - \gamma I \\
\frac{dR}{dt} = \gamma I
\end{cases}
$$

$$\beta = \text{transmission rate}$$

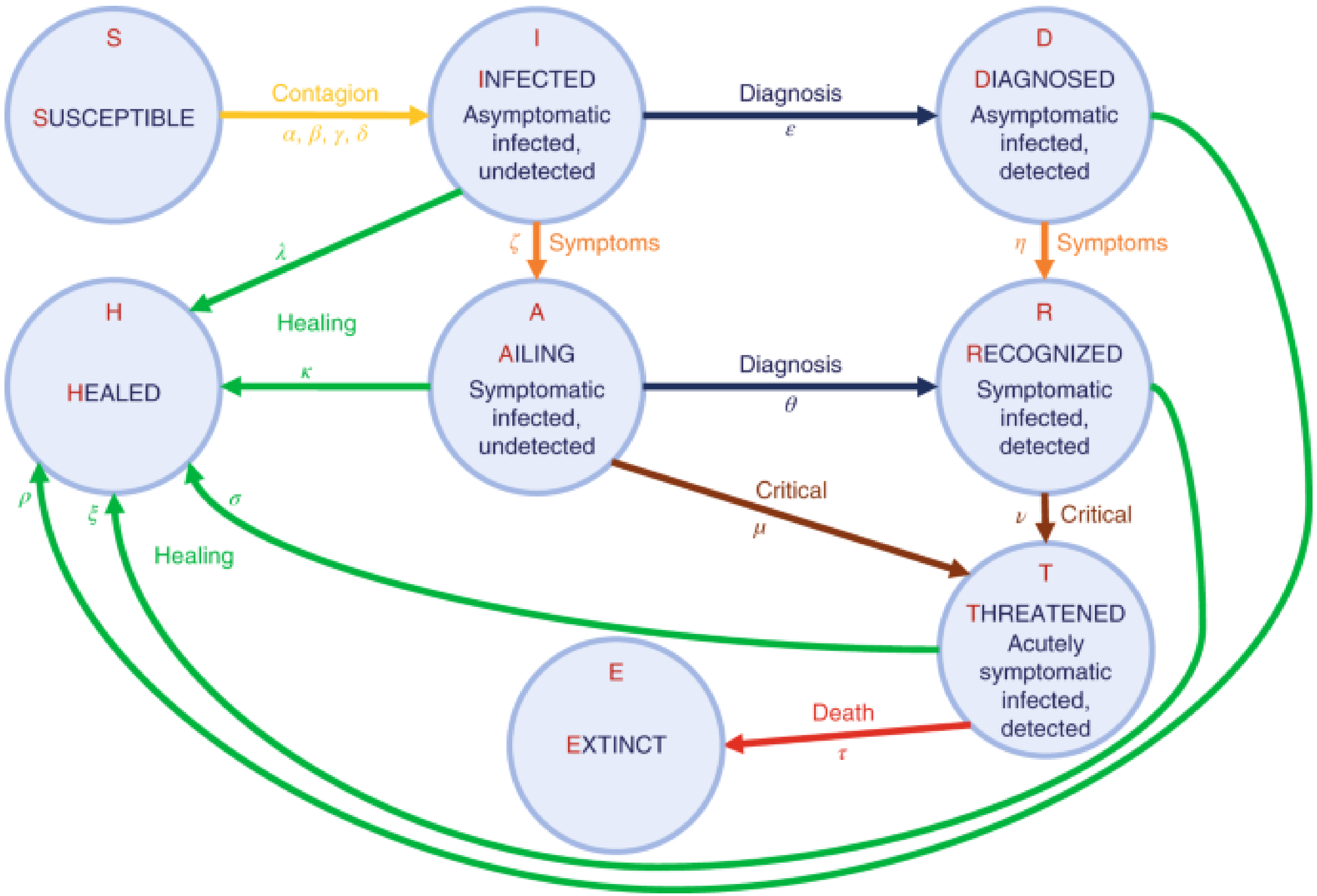




SIR model widely used

- Monitoring the progression of an epidemic
- Effective reproduction number compared to 1 → R_e
- Forecasting, how fast does it grow, is it receding?
- Public health policy : reducing the transmission

Adding new compartments ➡ many extensions, variants. Example:



Diffusion

- SIR : a description in a homogenous and isolated group
- How does the epidemic spread from a country to another, from a city to another, or even in neighborhoods?
- Diffusion
- Diffusion in epidemiology comes under several different guises

“As a matter of fact, all epidemiology, concerned as it is with the variation of disease from time to time or from place to place must be considered mathematically, if it is to be considered scientifically at all.”

Sir Ronald Ross

Nobel Price in Physiology 1902

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Sir Ronald Ross

Nobel Price in Physiology 1902

The variation of disease from time to time or from place to place

→ Partial Differential Equations (PDEs)

III. Spatial Diffusion of Epidemics

Non-local (spatial) spreading - Kendall (1965)

$$S = S(t,x), I = I(t,x), \quad x \in \mathbb{R}^N$$

$K(x,y) =$ Probability that individual at location *y* will infect individual at location *x*

$$\begin{cases} \partial_t S(t,x) = -\beta S(t,x) \int K(x,y) I(t,y) dy \\ \\ \partial_t I(t,x) = \beta S(t,x) \int K(x,y) I(t,y) dy - \gamma I(t,x) \end{cases}$$

Kendall was motivated by the study of rabies in Britain propagated by foxes

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Spatial variations

"from place to place"

Time variations

"from time to time"

Spatial diffusion with non-local spreading

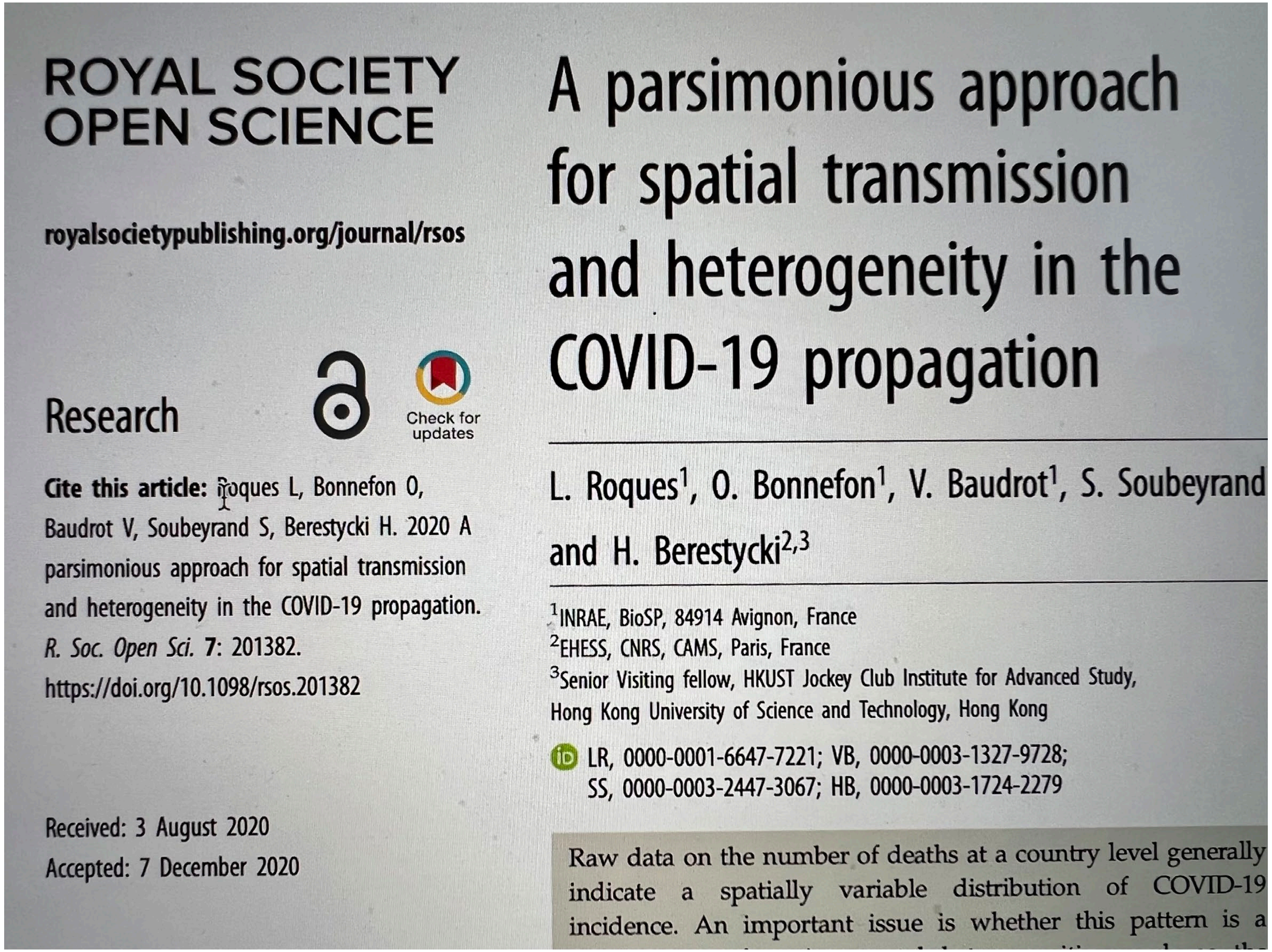
First model by Kendall involved non-local spreading (continuous)

A vast mathematical literature since

A model (discrete) with non-local transmission and applications to Covid-19 data in France:

L. Roques, HB et al., Royal Soc. Open Science (December 2020)

Spatial spreading



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Research

Check for updates

Cite this article: Roques L, Bonnefon O, Baudrot V, Soubeyrand S, Berestycki H. 2020 A parsimonious approach for spatial transmission and heterogeneity in the COVID-19 propagation. *R. Soc. Open Sci.* **7**: 201382. https://doi.org/10.1098/rsos.201382

Received: 3 August 2020
Accepted: 7 December 2020

A parsimonious approach for spatial transmission and heterogeneity in the COVID-19 propagation

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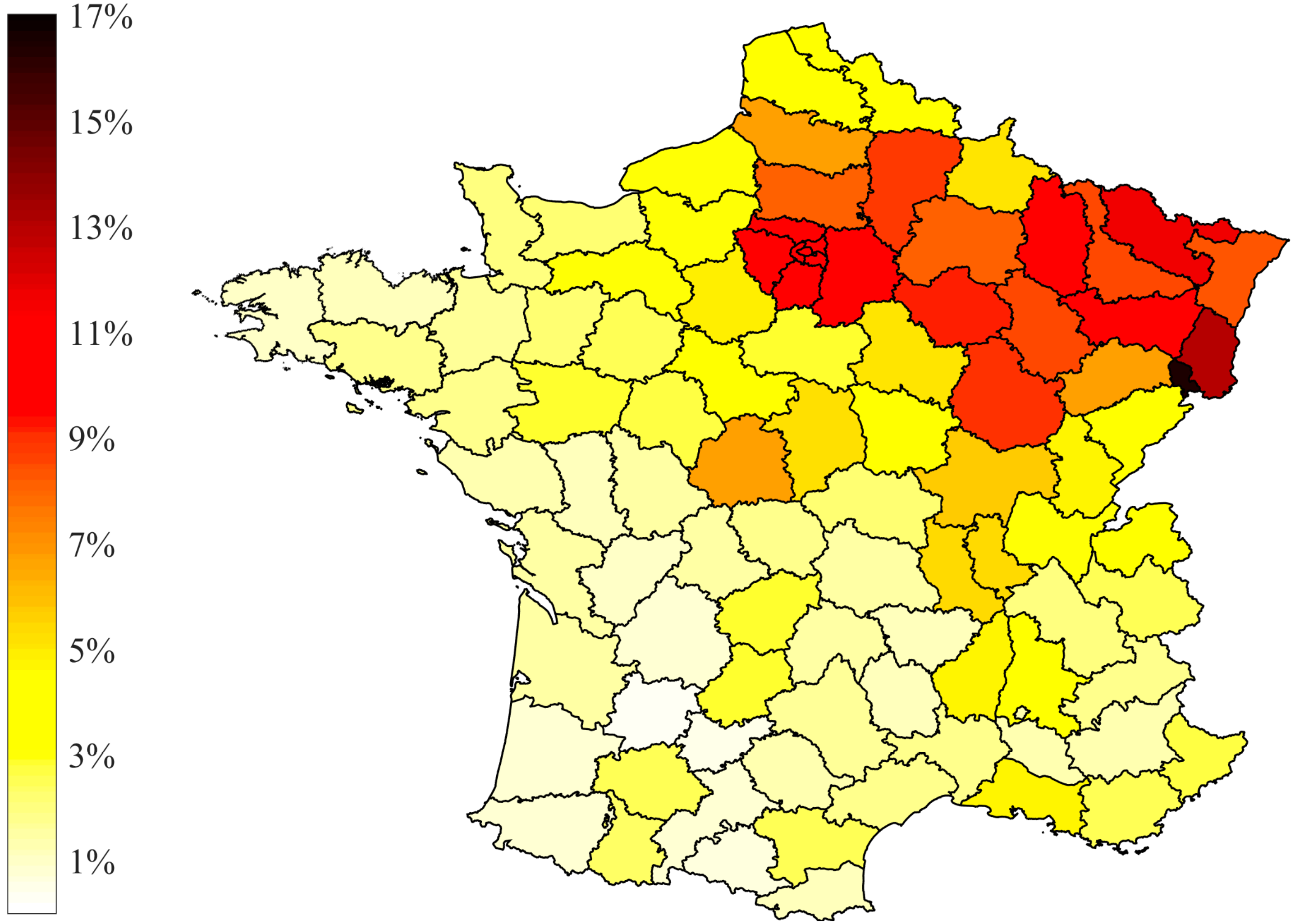
Raw data on the number of deaths at a country level generally indicate a spatially variable distribution of COVID-19 incidence. An important issue is whether this pattern is a

Inspired from earlier work on social contagion: https://www.nature.com/articles/s41598-017-18093-4

With L. Bonnasse-Gahot et al.

Propagation of the 2005 riots in France

Départements (counties) of France



Graph of n_d départements (counties) in France

$$S = S_k(t), \quad I = I_k(t), \quad R = R_k(t) \quad k = 1, \dots, n_d$$

$$\begin{cases} \partial_t S_k(t) = -\frac{\rho(t)}{N_k} S_k(t) \Sigma_{j=1}^{n_d} w_{j,k} I_j(t) \\ \partial_t I_k(t) = \frac{\rho(t)}{N_k} S_k(t) \Sigma_{j=1}^{n_d} w_{j,k} I_j(t) - \gamma I_k(t) \\ \partial_t R_k(t) = \gamma I_k(t) \end{cases}$$

Analogous to →

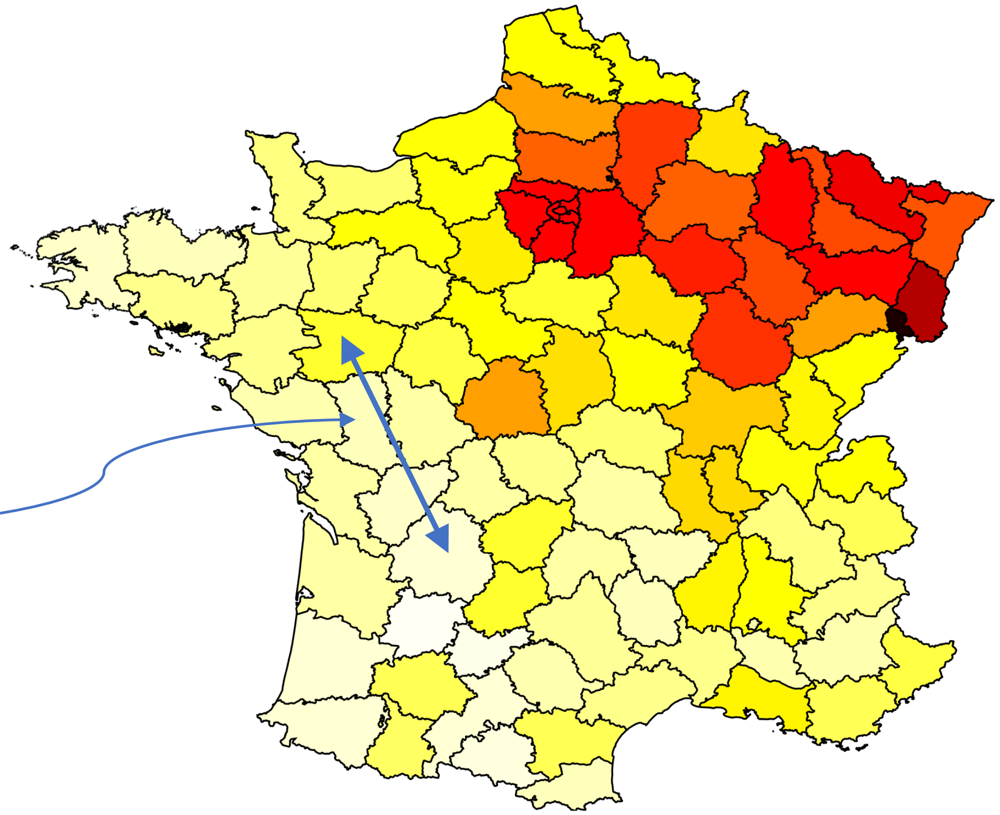
$$\begin{cases} \partial_t S(t,x) = -\beta S(t,x) \int K(x,y) I(t,y) dy \\ \partial_t I(t,x) = \beta S(t,x) \int K(x,y) I(t,y) dy - \gamma I(t,x) \end{cases}$$

Weights $w_{j,k}$

- Power law decay in the distance
- dist(j,k) : distance from centroids of counties j and k
- d_0 a scale parameter
- d_0 and δ global parameters

$$w_{j,k} = \frac{1}{1 + (\text{dist}(j,k)/d_0)^{\delta}}$$

Distance between two counties



$\mathrm{dist}(j, k)$

Incidence rate

- Simulation of the model
- Localized initial cases
- Evolution January - December 2020
- Public health measures (lockdown) reflected in coefficient $\rho(t)$
- Darker colors = higher incidence rates

Using the model

- Good qualitative agreement with the observed spatial dynamics of the COVID epidemics in France
- Few parameters to fit globally
- Use for forcasting
- Study the impact of public health measures
- In particular effects of
  - Lockdown to reduce transmission rate
  - Travel restrictions

Testing effects of measures : Limiting movement vs limiting transmission per contact

Comparing 4 strategies

1. No restrictions

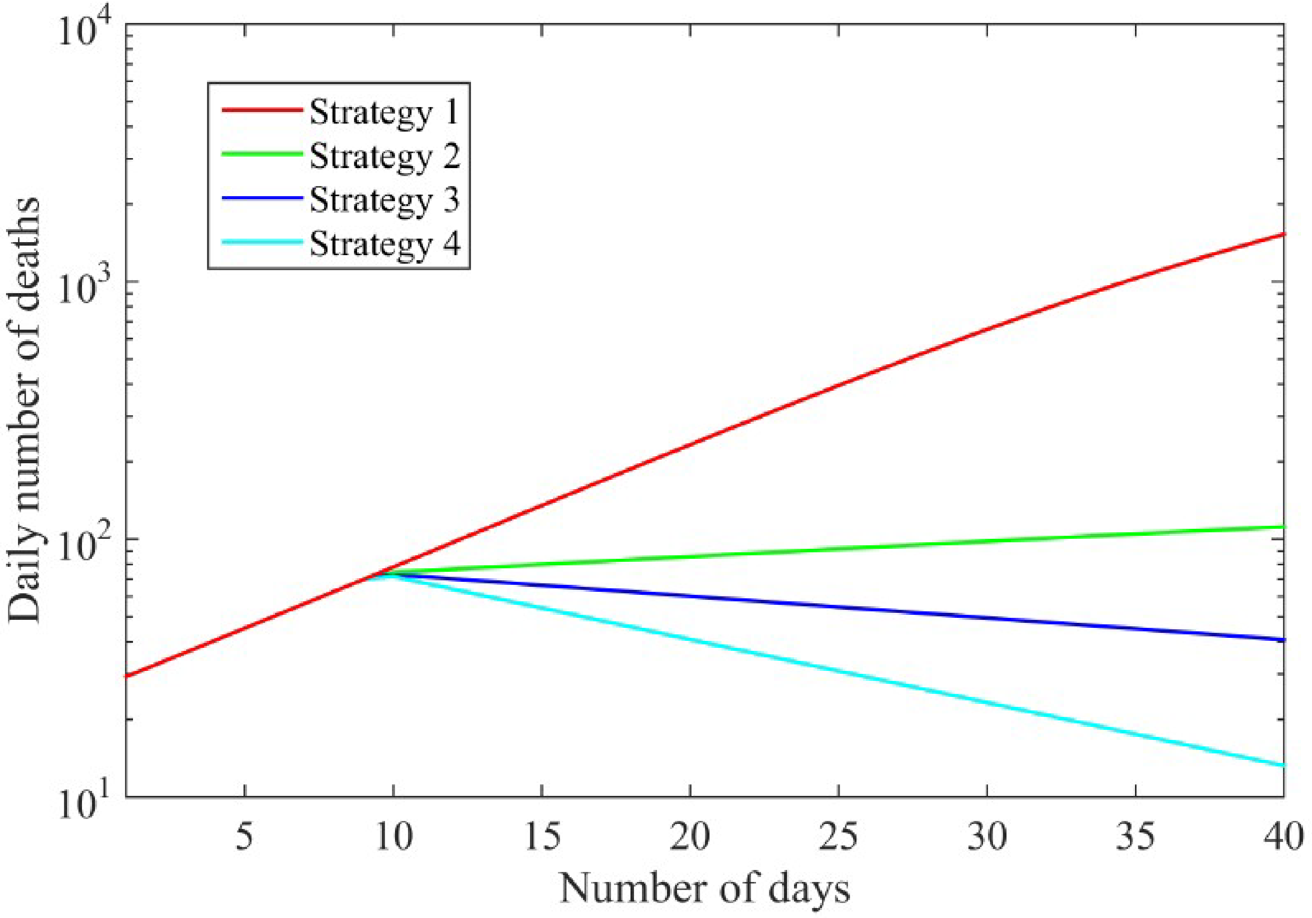
2. Restriction on intercounty travel

3. Reduction of contact rate at national level (social distancing etc.)

4. Reduction of contact rate *and* restriction on intercounty travel

Testing strategies Forecasting the effects in daily number of deaths in France

1. No restrictions
2. Restricting intercounty travel
3. Reducing contact rate (social distancing etc.)
4. Reducing contact rate *and* restricting intercounty travel



Logarithmic scale

)aily number of deaths due to a new outbreak in logarithmic scale; comparis
;ement strategies. The number of deaths is computed over the whole countr

Local diffusion in epidemiology: spatial spreading Random movement of susceptible and infected

$$S = S(t,x), I = I(t,x), \quad x \in \mathbb{R}^N$$

$$\begin{cases} \partial_t S - \mu_S \Delta S = -\beta S I \\ \partial_t I - \mu_I \Delta I = +\beta S I - \gamma I \end{cases}$$

Variations in time

Spatial random movement of individuals

Activity/susceptibility models
a general class of models

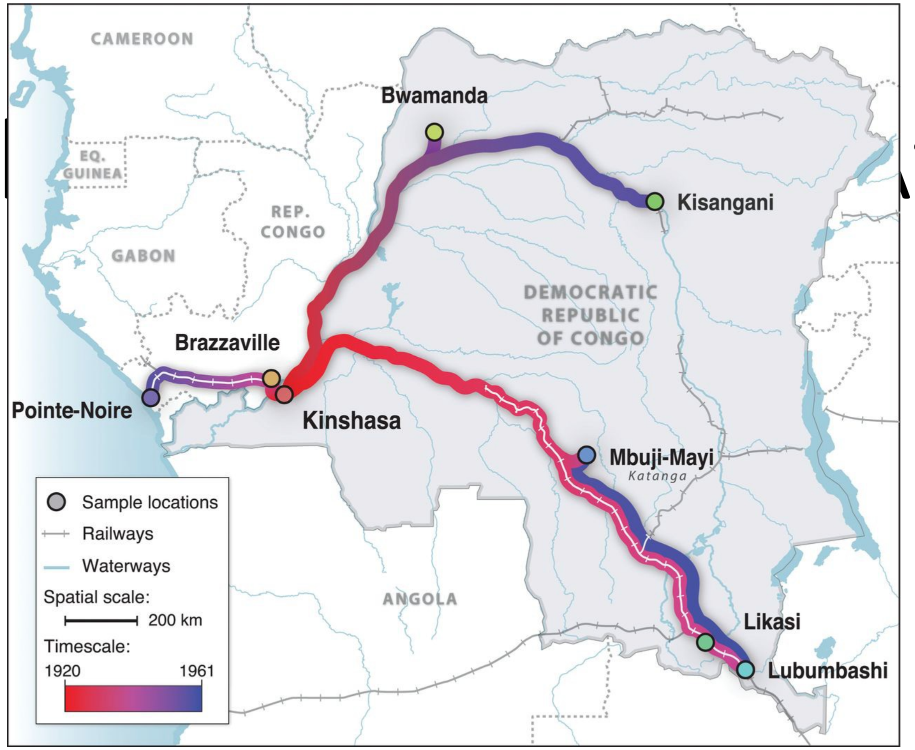
- Many different type of models : predator - prey, riots, social behavior,...
- HB, S Nordmann, L. Rossi : *Modeling the propagation of riots, collective behaviors, and epidemics,*

Mathematics in Engineering
2022, Volume 4, Issue 1: 1-53. doi: 10.3934/mine.2022003

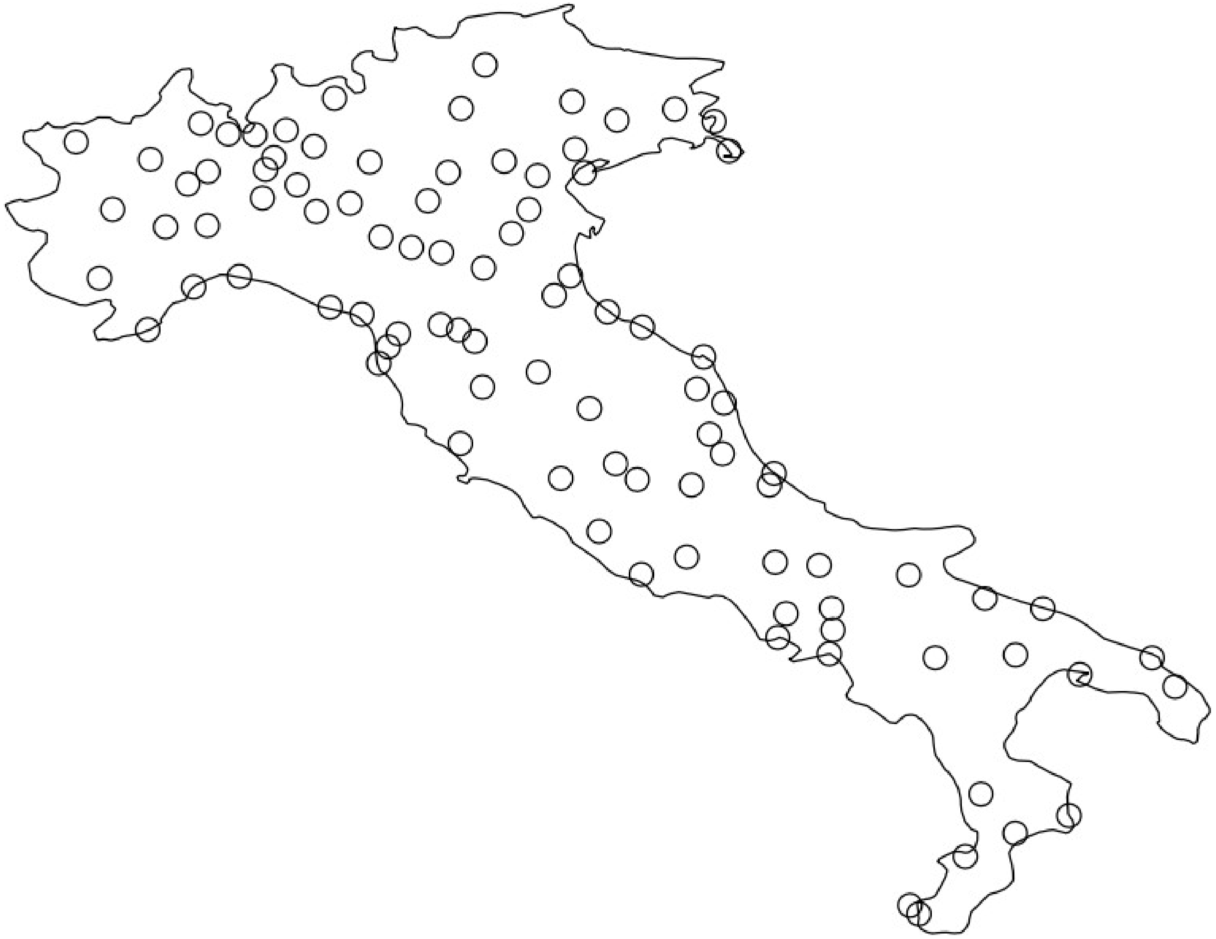
IV. Propagation of epidemics along roads

(roads, railways, waterways…)

Earl … frica

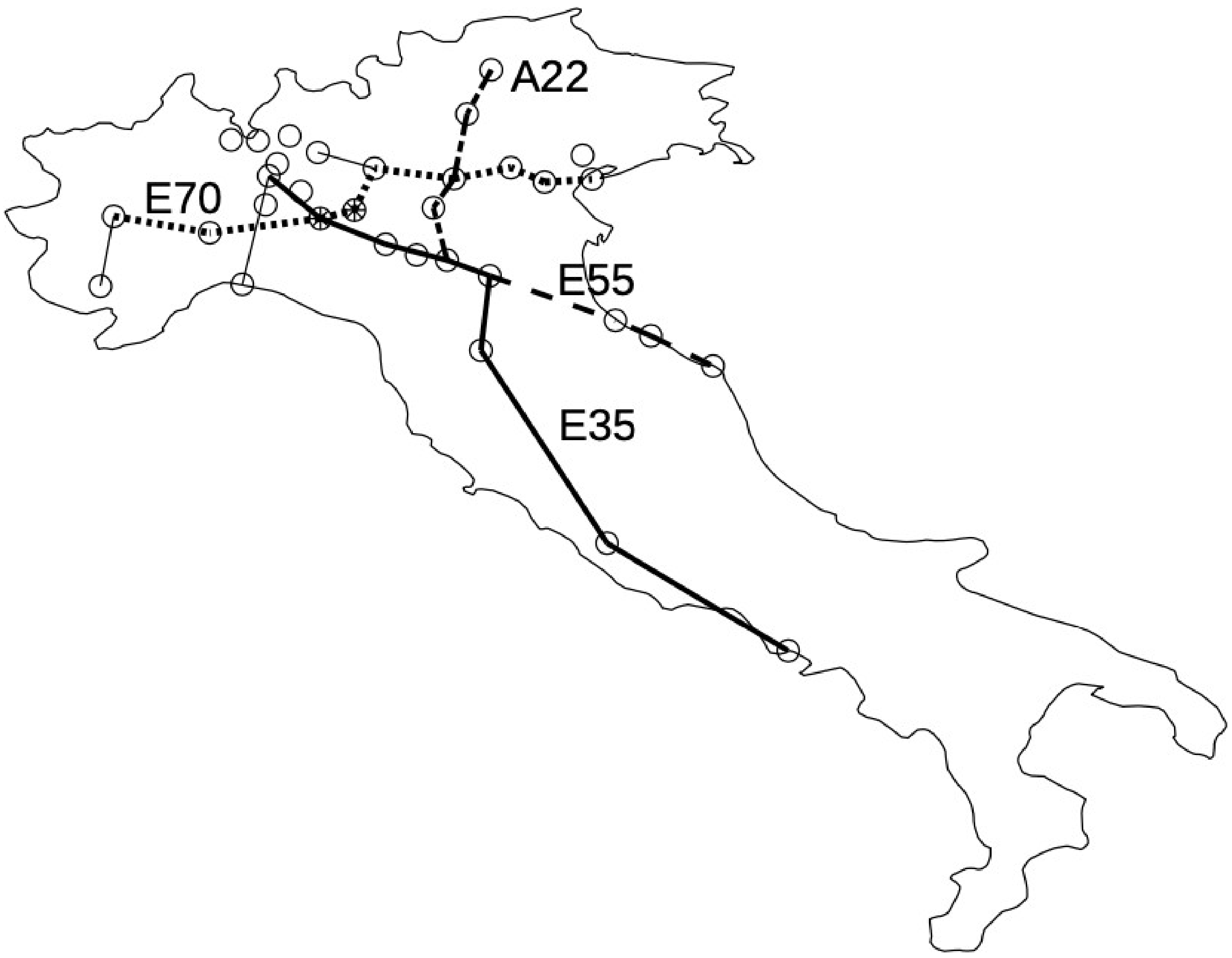


[Faria et al., *The early spread and epidemic ignition of HIV-1 in human populations*, Science (2014)]



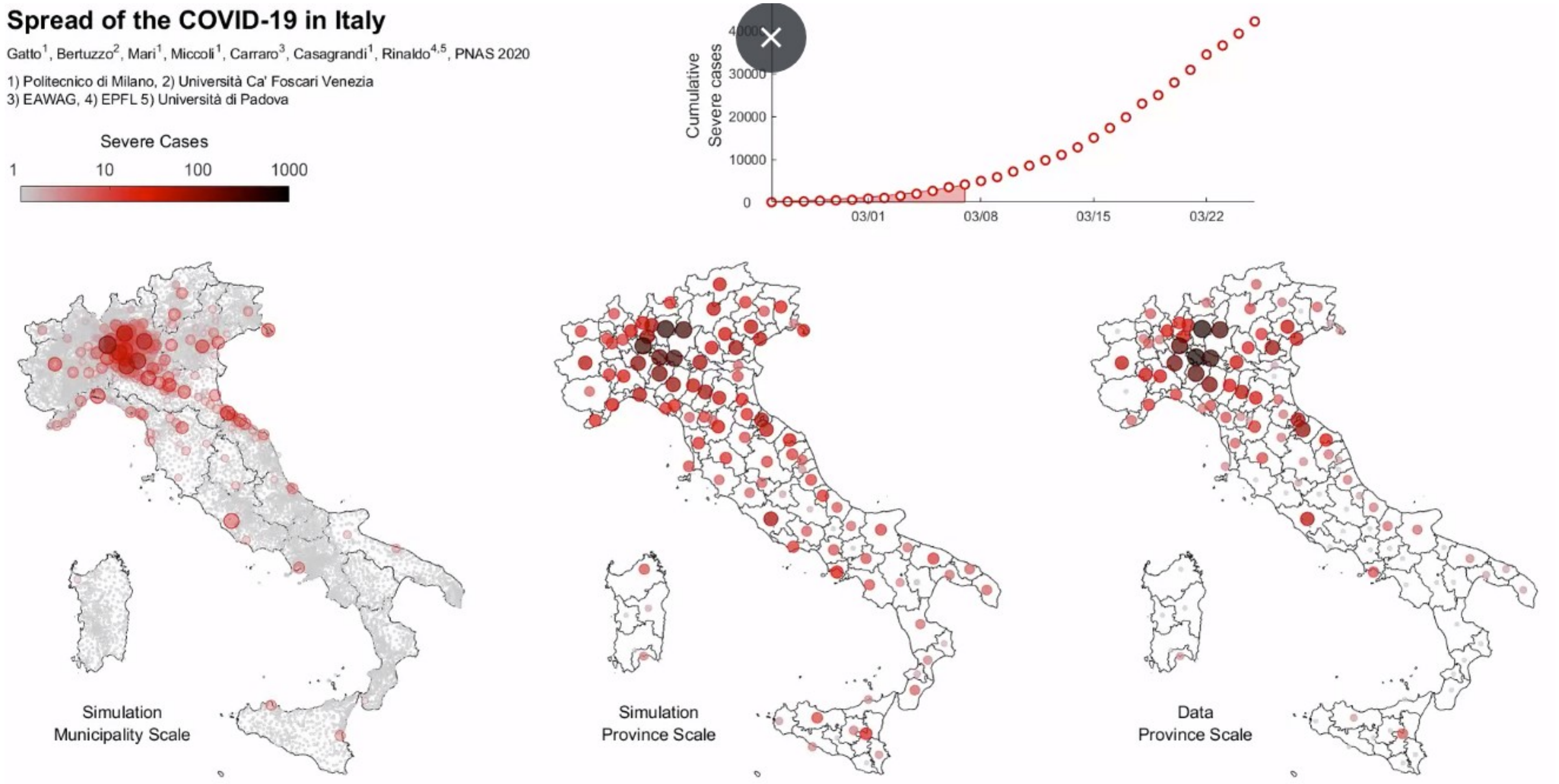
[*Sebastiani et al.*, Il Coronavirus ha viaggiato in autostrada (2020)]

Propagation of covid-19 in Italy



Sebastiani et al., The Coronavirus travelled on expressways (2020)

Propagation of covid-19 in Italy



[*Gatto et al.*, PNAS (2020)]

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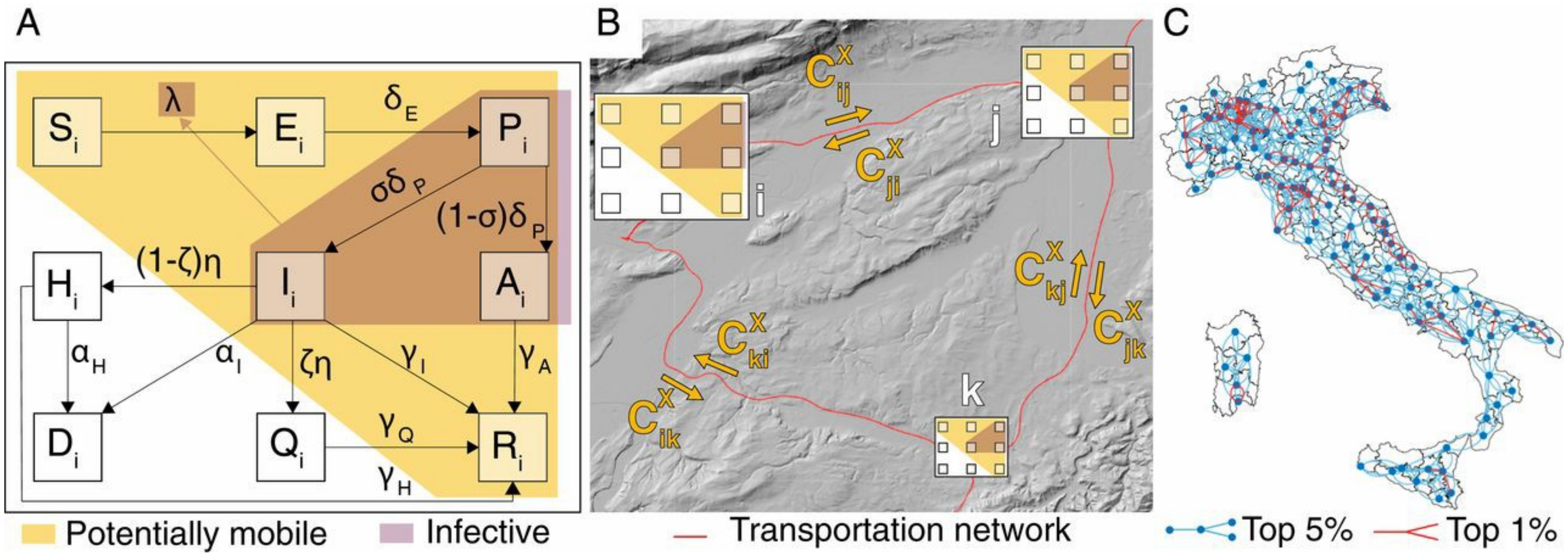

[*Gatto et al.*, PNAS (2020)]

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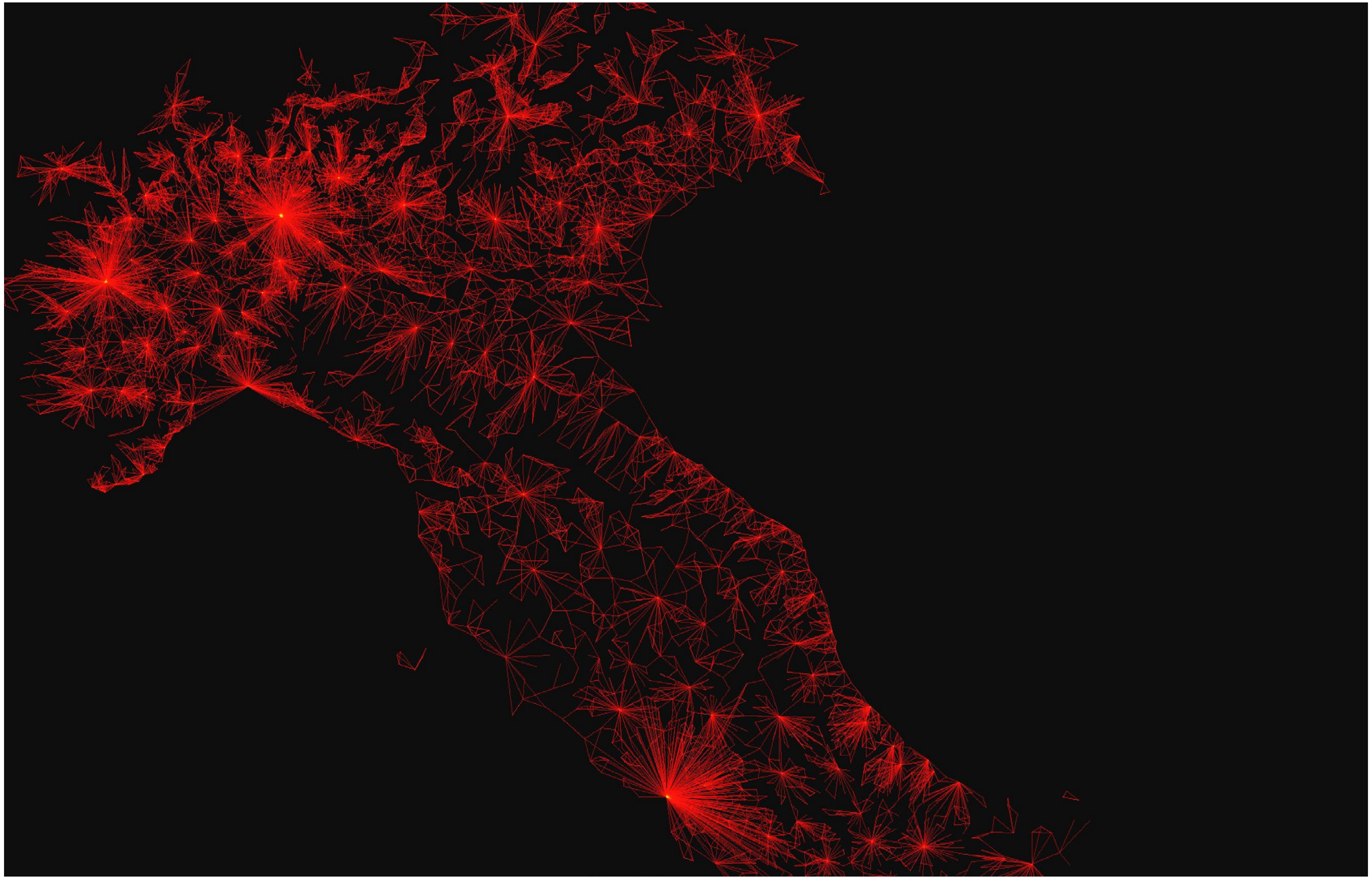


[*Gatto et al.*, PNAS (2020)]

Diffusion in epidemiology: Rapid diffusion along roads

- Berestycki, H., Roquejoffre, JM. & Rossi, L.

Propagation of Epidemics Along Lines with Fast Diffusion.

Bull Math Biol **83**, 2 (2021).

https://doi.org/10.1007/s11538-020-00826-8

The SIRT model

- Large population, domain : a half plane limited by a line.
  - Susceptibles $\to$ Infectives in the domain.
  - A new compartment : Travelling Infectives on the line.
- I contaminate S at constant rate until removed.
- Infectives
  - move in the domain, travel on the line.
- Line and plane exchange infectives at constant rate.

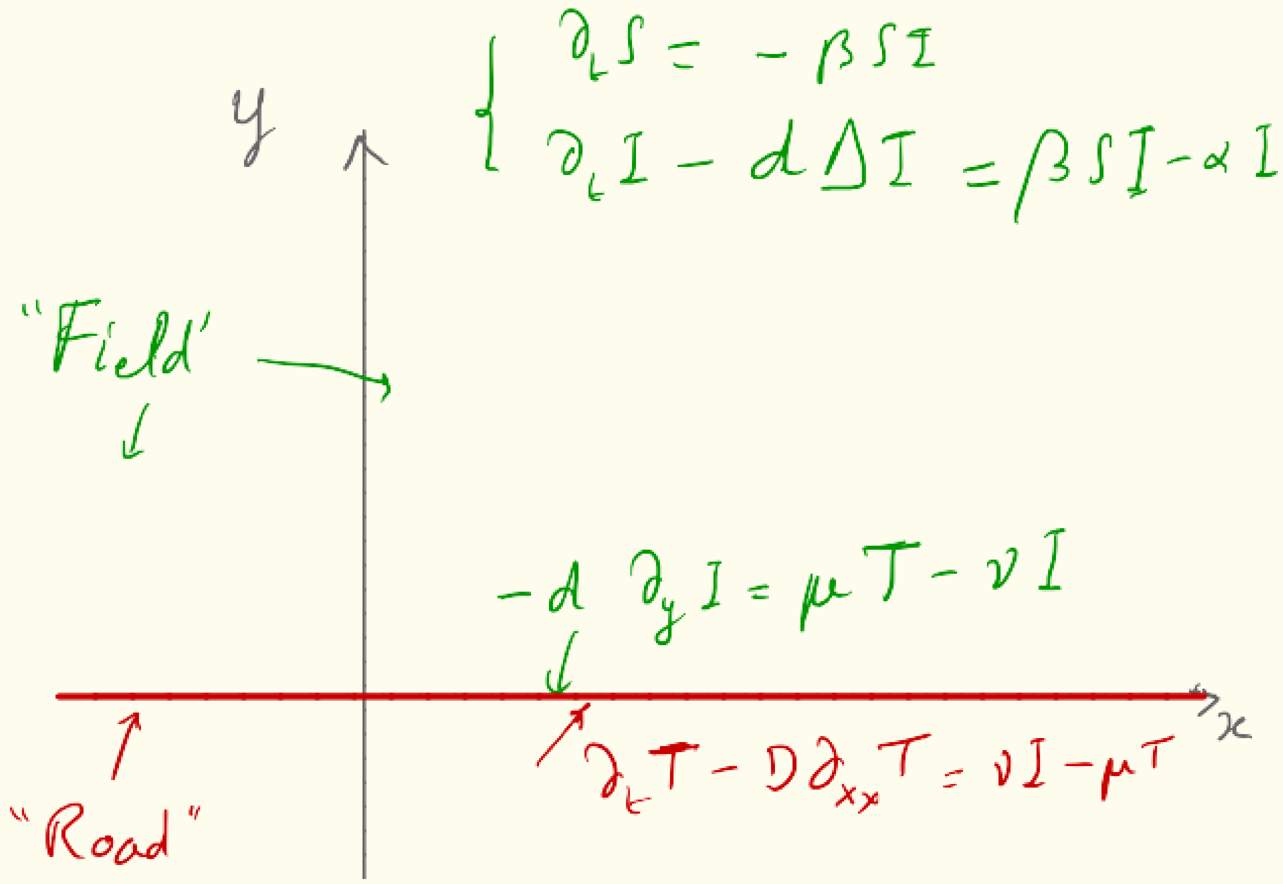
- At $t = 0$ a (small) density of infectives is introduced

Model in the half-space $\{y > 0\}$

- **The line :** $(x, 0)$, $x \in \mathbb{R}$.
- **Unknowns :**
  - Susceptibles/Infective : densities $S(t, x, y)$, $I(t, x, y)$.
  - $T(t, x)$: density of travelling infectives.
- **Parameters :**
  - S_0 : initial density of susceptibles.
  - α : removal rate of infectives, β : transmission rate of infection.
  - μ : transmission rate between line and half plane.
  - ν : transmission rate between half plane and line.
  - d : diffusion in the domain, D : diffusion on the line.

Basic reproduction number : $R_0 = \dfrac{S_0 \beta}{\alpha}$

The SIRT model in a half-plane



Spreading speed

- If $c > c^T_{SIR}$, then $(u(t,x), v(t,x,y)) \to 0$ if $|x| \geq ct$.
- If $c < c^T_{SIR}$, then $(u(t,x), v(t,x,y)) \to (u^*, v^*)$ if $|x| \leq ct$.

1. If $D \leq 2d$, then $c^T_{SIR} = c_{SIR}$.
2. If $D > 2d$, then $c^T_{SIR} > c_{SIR}$.

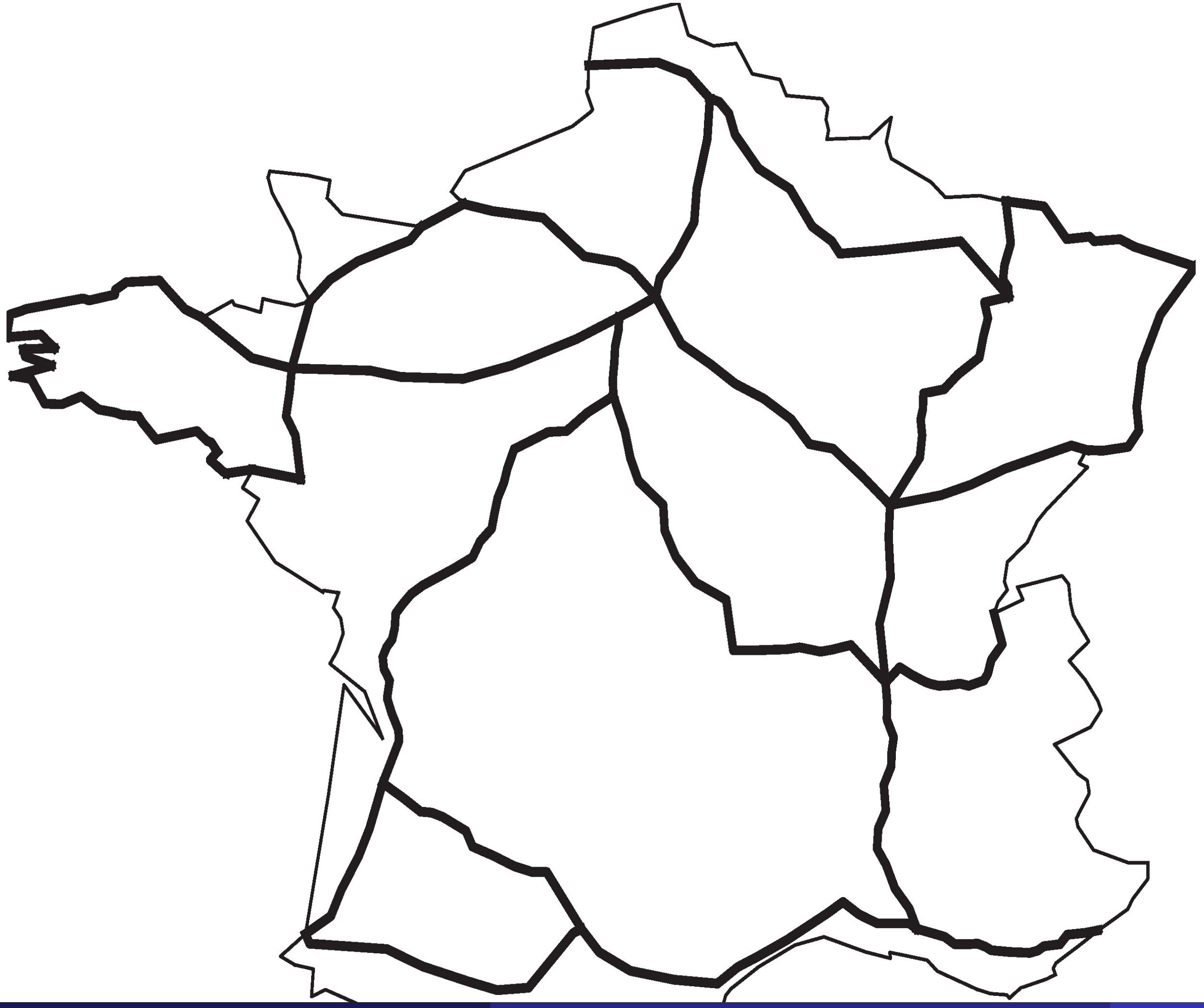
If x is large, $I(t,x,y)$ peaks around $t \sim \dfrac{x}{c^T_{SIR}}$.

An important outcome:

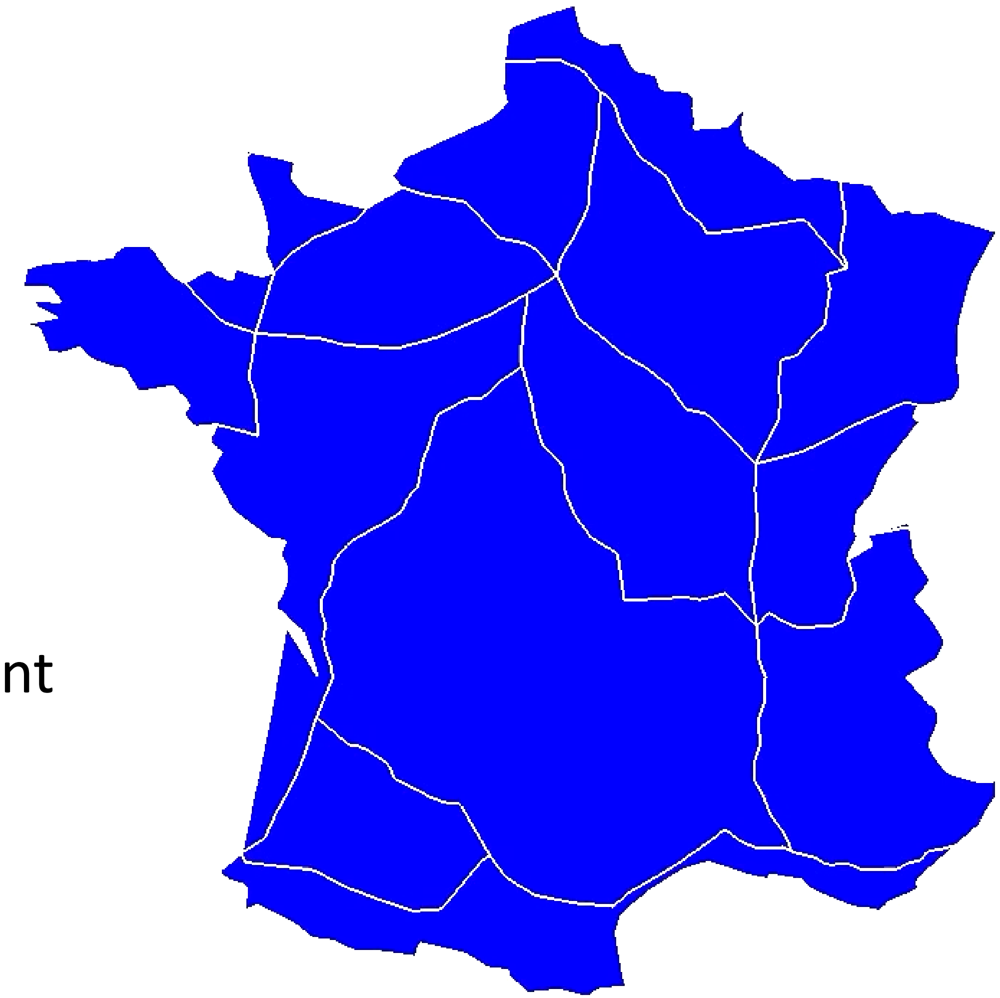
Propagation may be fast even if R_0 close to 1 !

Simulation : France with major expressways

Epressways : traffic≥15 000 cars / day.



Simulation of COVID propagation in France using the SIRT model with major roads

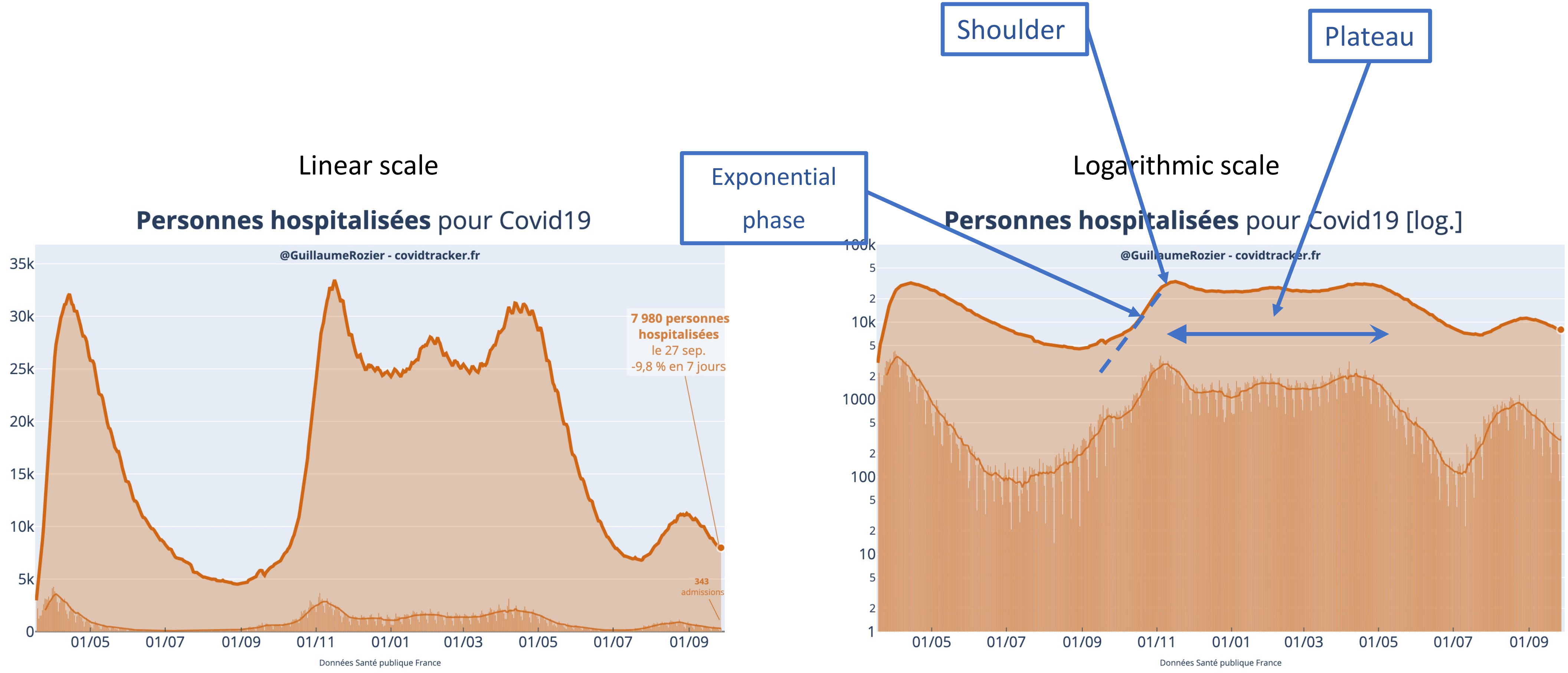


Good qualitative agreement with observed spreading of covid

Avant 15 mars

V. Social Diffusion

Observed complex dynamics : **Epidemic plateaus**
Hospitalized (7 days moving average)



Los Angeles Times

SCIENCE

Coronavirus Today: Have we finally plateaued?

By AMINA KHAN | STAFF WRITER

JAN. 19, 2021 7:16 PM PT

The Washington Post

Democracy Dies in Darkness

Middle East
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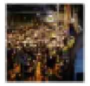

National
● **Live updates:** Covid-19 live updates: As restrictions drop, new coronavirus infections in ...

Europe
AstraZeneca's U.S. trial show coronavirus vaccine is 79 per effective

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Local

Coronavirus cases plateau across D.C. area as region's caseload surpasses 200,00

The New York Times

In France, Covid cases plateau, vaccinations still lag and weariness sets in.

Mirror

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Covid cases, hospitalisations and deaths all 'plateauing and starting to fall'

Chief Scientific Advisor Patrick Vallance said the UK remained in a 'difficult position' - as Boris Johnson extended lockdown for another three weeks

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By Oliver Milne Political Correspondent
17:33, 27 JAN 2021 | UPDATED 19:14, 27 JAN 2021

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U.S. NEWS JANUARY 21, 2021 / 10:37 PM / UPDATED 2 MONTHS AGO

Fauci says U.S. coronavirus infections may be plateauing, vaccines should protect against new variants

By Jeff Mason, Carl O'Donnell 2 MIN READ

WASHINGTON (Reuters) - Dr. Anthony Fauci, the top U.S. infectious disease expert, said on Thursday that based on recent seven-day averages, coronavirus infections may be about to hit a plateau in the United States.

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Es gehe darum, "noch mehr" zu tun, um die Infektionszahlen zu senken, sagte Seibert. Mit Hilfe der geltenden einschneidenden Maßnahmen sei es zwar geschafft worden, "die Zahlen auf einem gewissen Plateau zu halten". Nach wie vor sei Deutschland aber weit von dem Ziel entfernt, den sogenannten Inzidenzwert wieder unter 50 Infektionen pro 100.000 Einwohner in sieben Tagen zu senken.

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Why are there plateaus ?

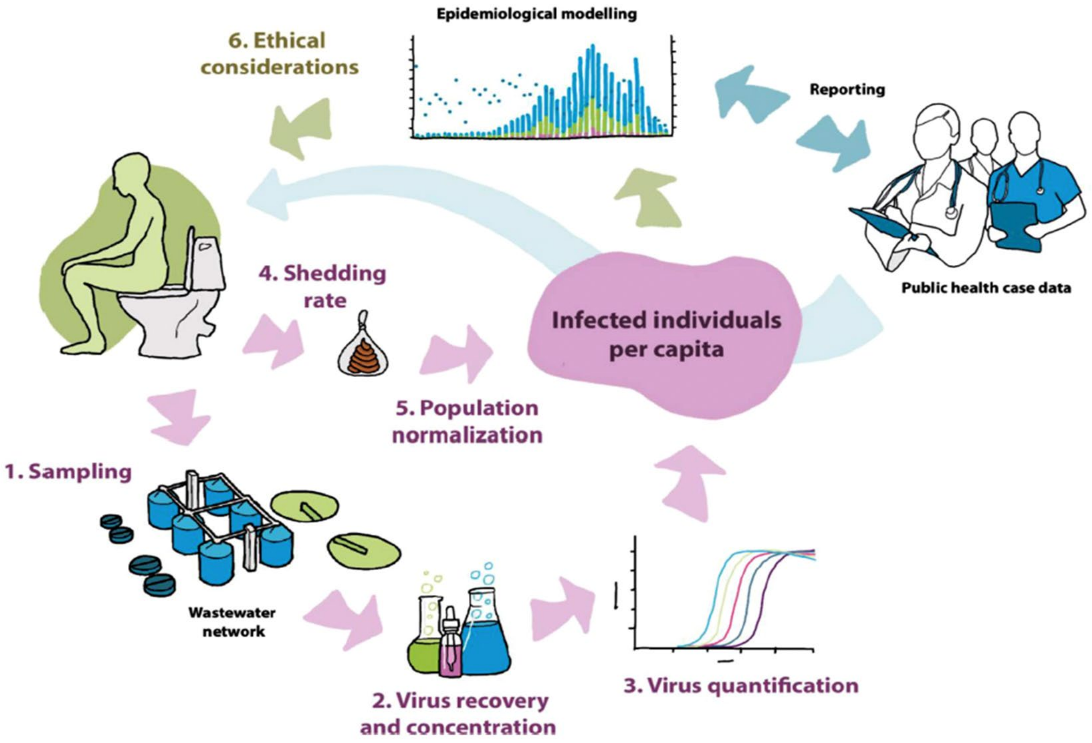
Complex dynamics of epidemics

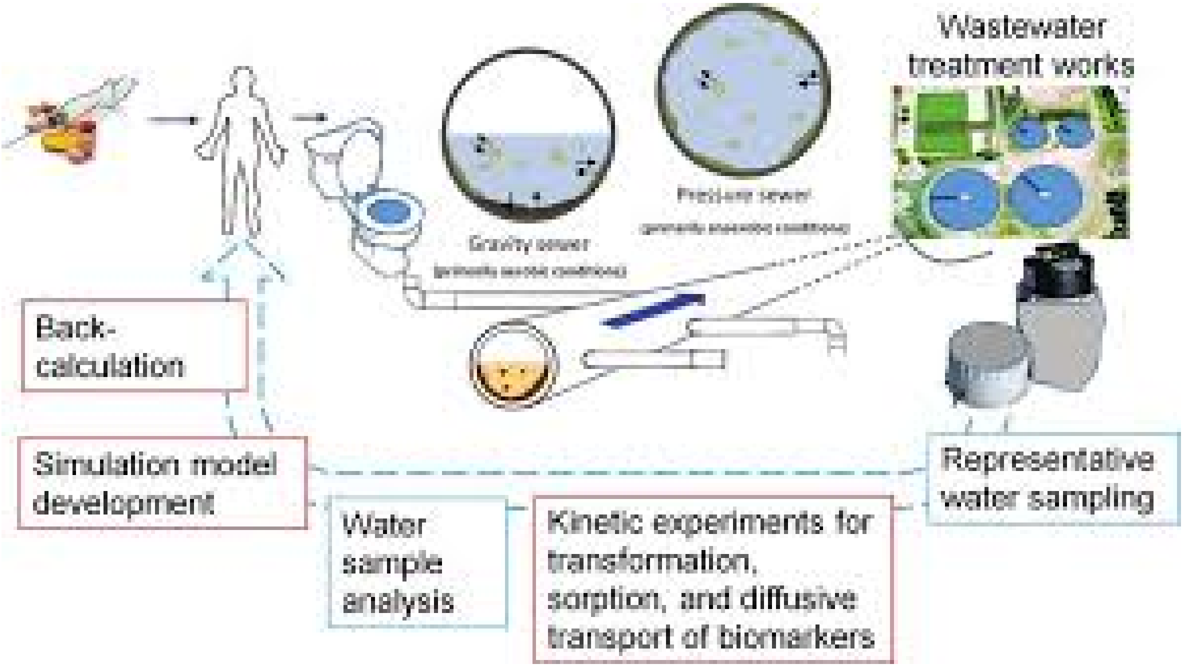
Starting with an observation from wastewater measures in South of France

Why are there plateaus and rebounds ?

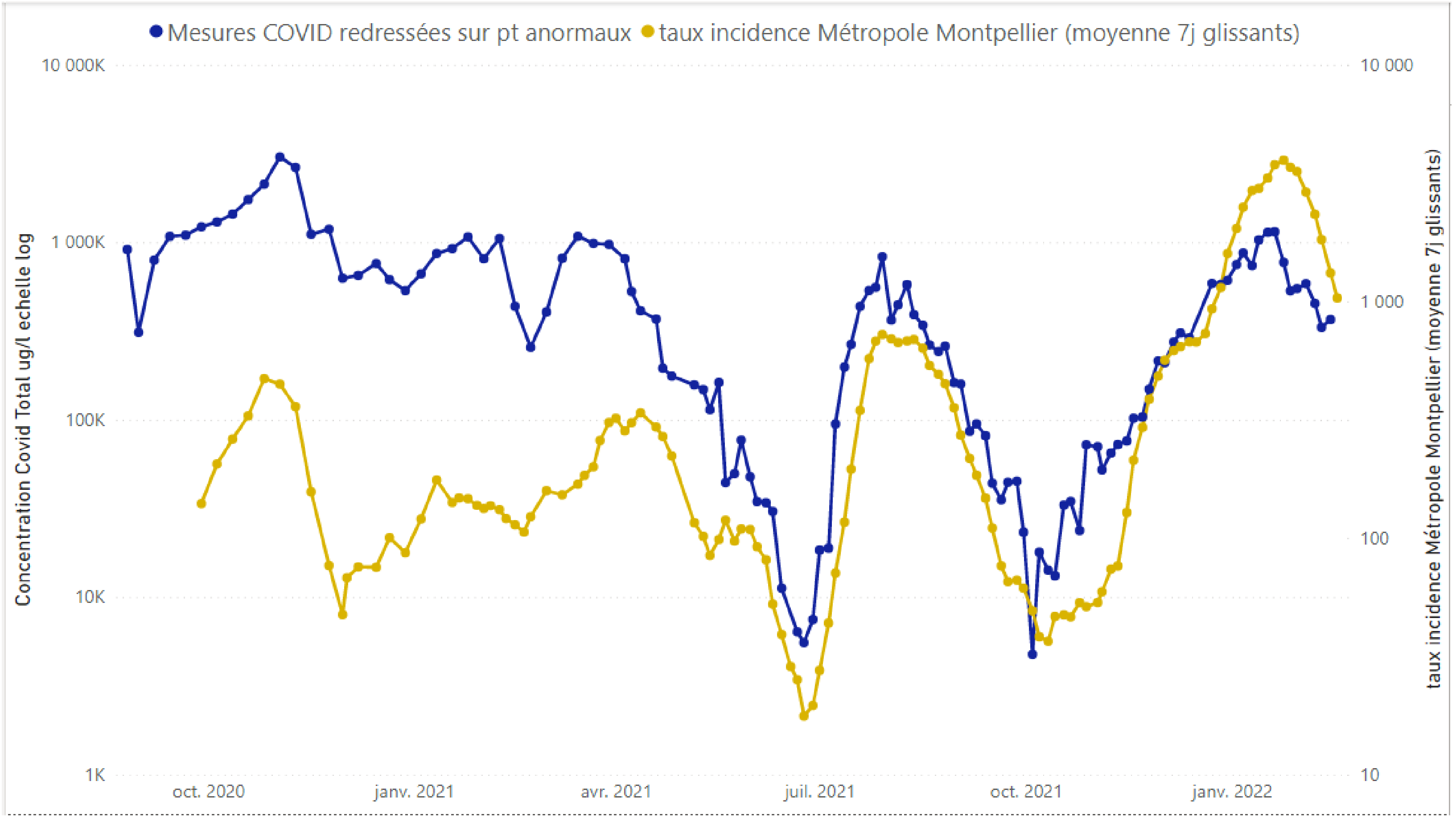
- Effects of public health policy ?
- Effects of *awareness* ?
- Effects of *fatigue ?*
- We claim that plateaus, rebounds etc. result from the *intrinsic dynamics* of epidemics
- Observations from Wastewater based epidemiology (WBE)
- WBE: game changer in epidemiology,
  - New possibilities
  - Many challenges, including mathematical

Wastewater Based Epidemiology

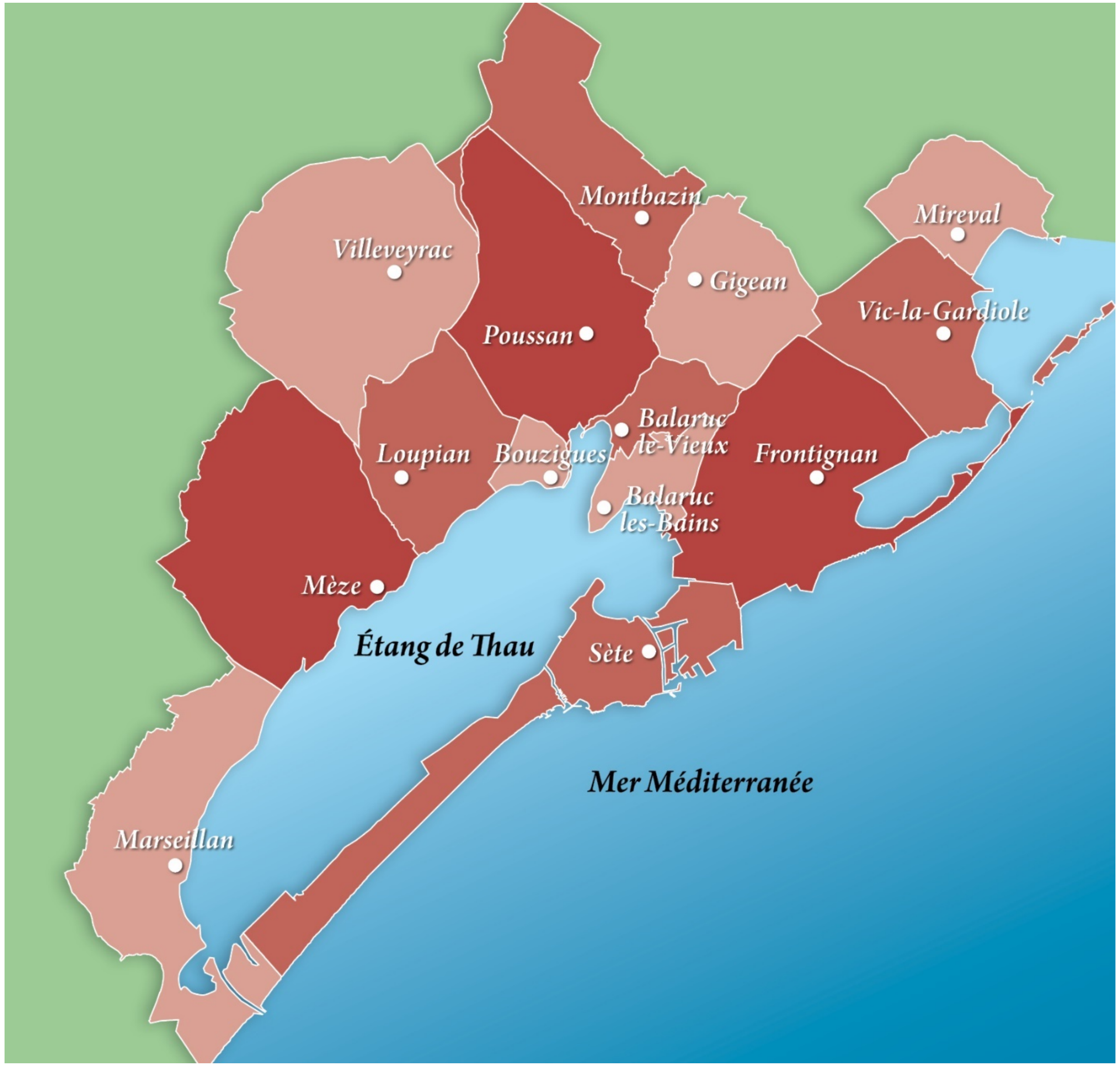




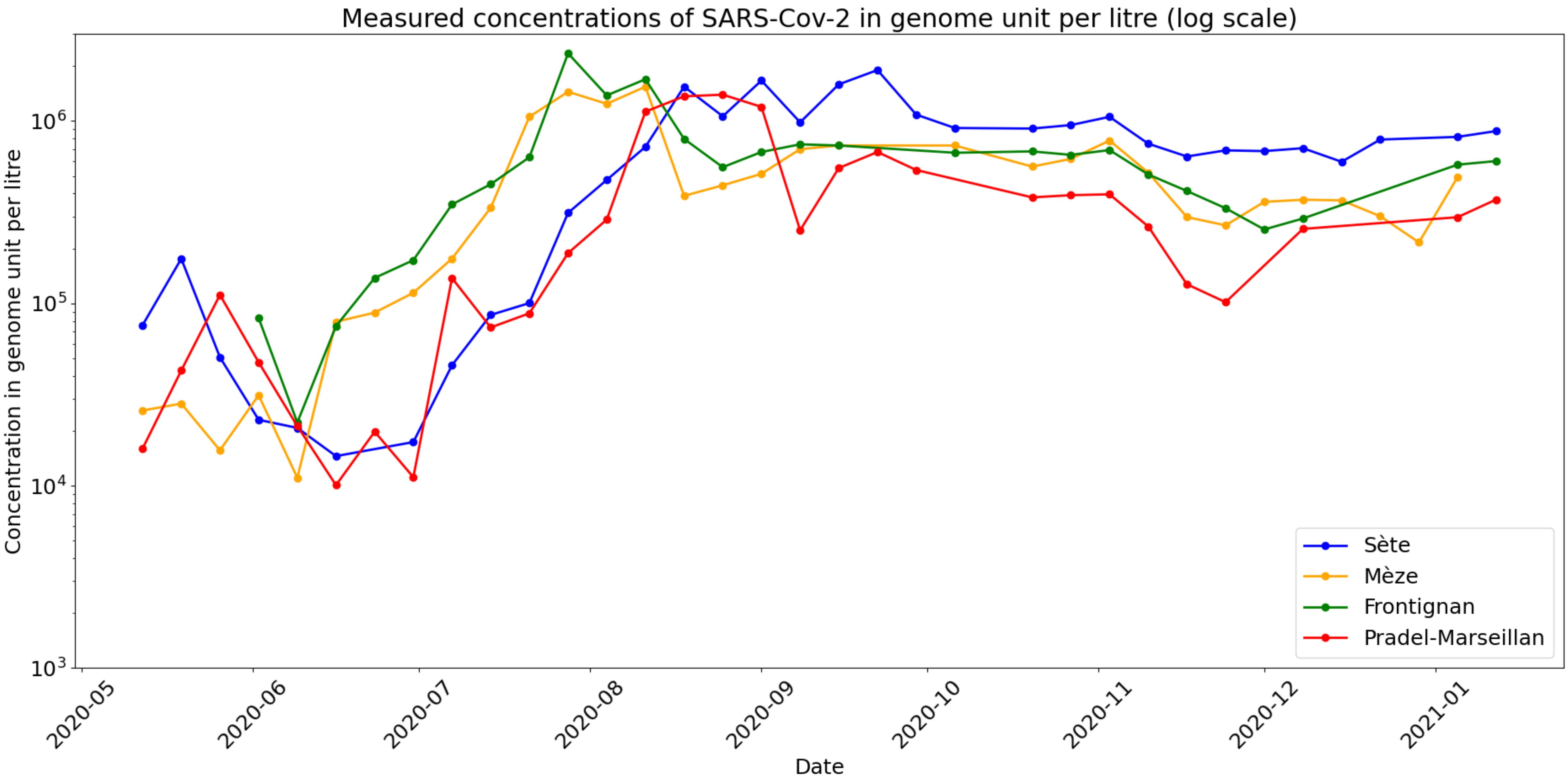
Measures in Montpellier



Thau lagoon map



Wastewater based epidemiology: the Thau lagoon experiment



A new model involving diffusion: Heterogeneous behaviors and Social Diffusion

AIM TO UNDERSTAND SOME COMPLEX FEATURES IN EPIDEMICS

IN PARTICULAR, *PLATEAUS* AND *REBOUNDS*

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Plateaus, rebounds and the effects of individual behaviours in epidemics

Henri Berestycki[1,2]✉, Benoît Desjardins[3,4], Bruno Heintz[4] & Jean-Marc Oury[4]

Plateaus and rebounds of various epidemiological indicators are widely reported in Covid-19 pandemics studies but have not been explained so far. Here, we address this problem and explain the appearance of these patterns. We start with an empirical study of an original dataset obtained from highly precise measurements of SARS-CoV-2 concentration in wastewater over nine months in several treatment plants around the Thau lagoon in France. Among various features, we observe that the concentration displays plateaus at different dates in various locations but at the same level. In order to understand these facts, we introduce a new mathematical model that takes into account the heterogeneity and the natural variability of individual behaviours. Our model shows that the distribution of risky behaviours appears as the key ingredient for understanding the observed temporal patterns of epidemics.

Epidemiological model with heterogeneous behavior and social diffusion

- $a \in (0,1)$ defines an indicator of risky behavior of susceptibles
- The infection transmission rate $a \mapsto \beta(a)$ is an increasing function of a
- $S(t,a)$ the density of individuals at time t associated with risk parameter a
- $I(t)$ the total number of infected
- $R(t)$ the number of removed individuals

- γ^{-1}: the inverse of typical duration (in days) of the disease
- d a positive diffusion coefficient

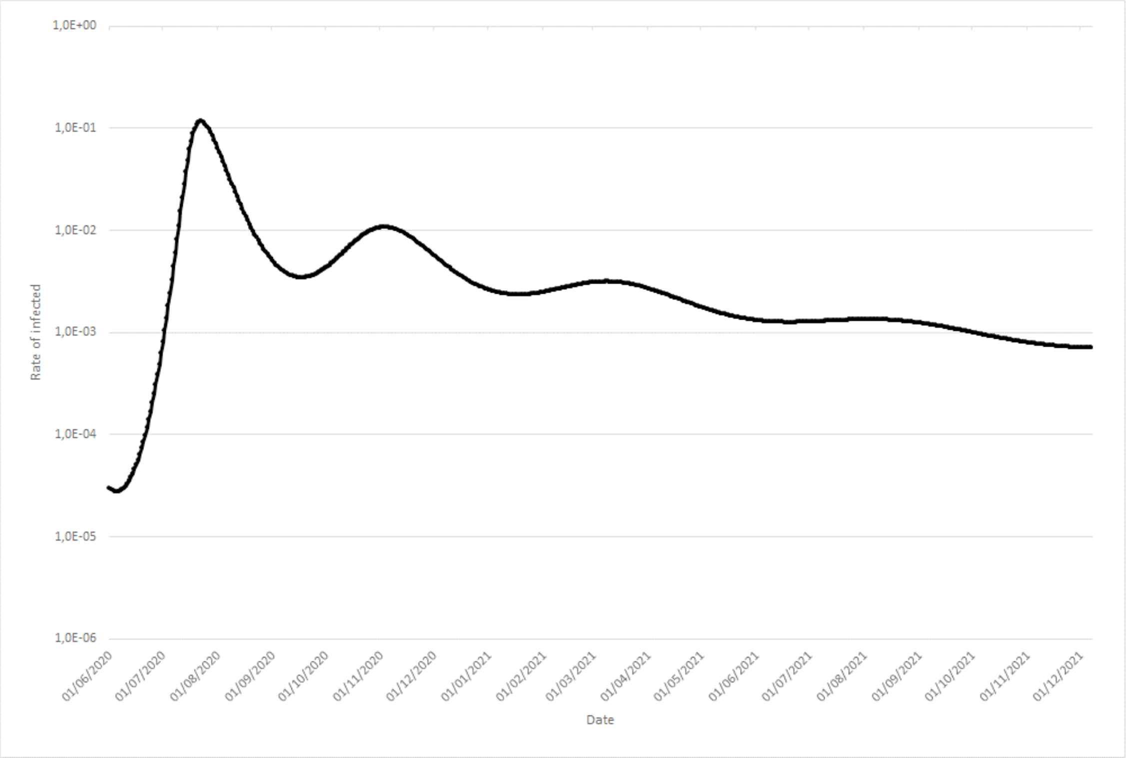
PDE model with heterogeneous behavior and social diffusion

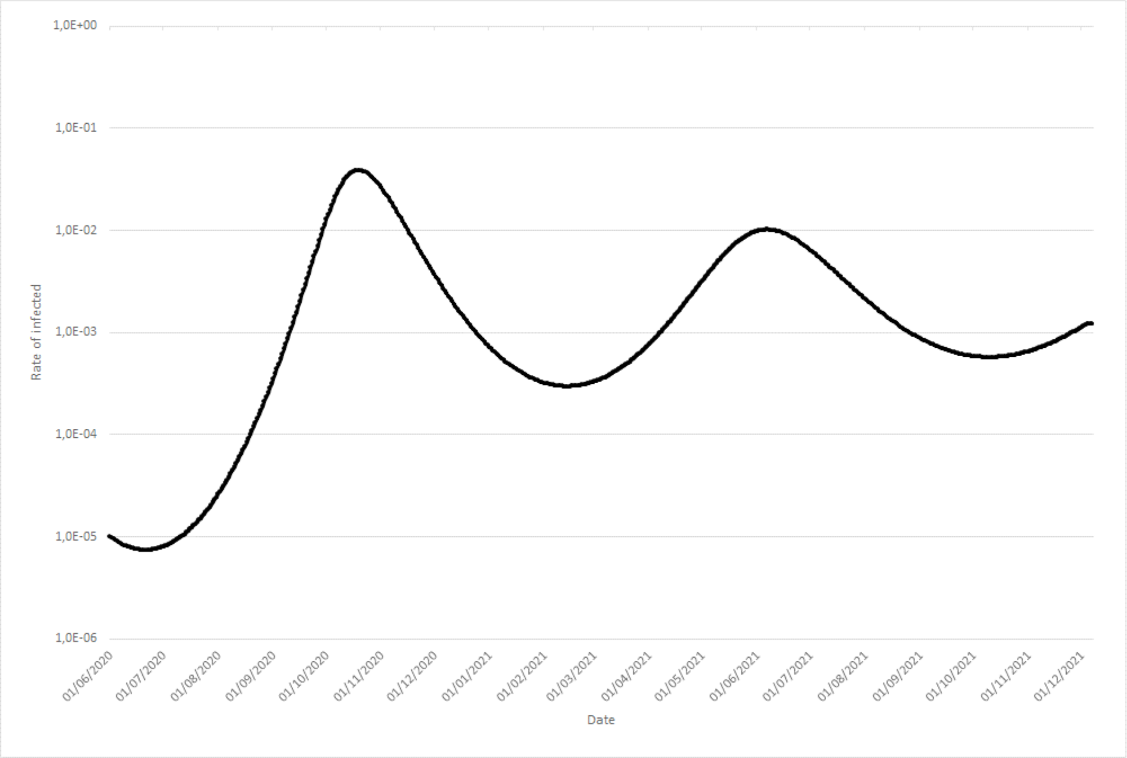
$$\partial_a S(t,0) = \partial_a S(1,0) = 0 \quad \text{if} \quad d > 0.$$

$$\begin{cases} \dfrac{\partial S(t,a)}{\partial t} = d\,\dfrac{\partial^2 S(t,a)}{\partial a^2} - \beta(a)S(t,a)\dfrac{I(t)}{N}, \\ \dfrac{\mathrm{d}I(t)}{\mathrm{d}t} = \dfrac{I(t)}{N}\displaystyle\int_0^1 \beta(a)S(t,a)\,da - \gamma I(t), \\ \dfrac{\mathrm{d}R(t)}{\mathrm{d}t} = \gamma I(t), \end{cases}$$

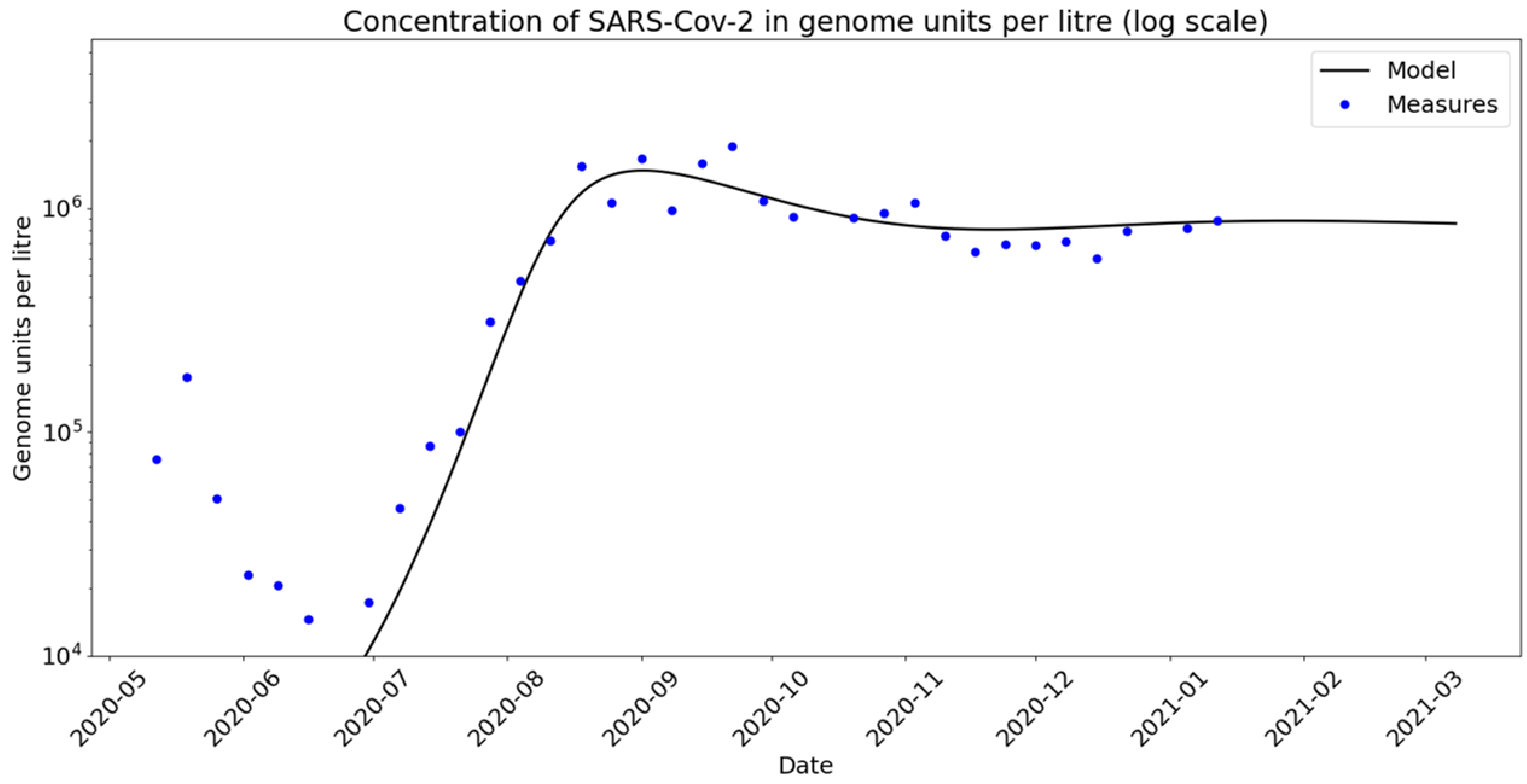
$$S(0,a) = S_0(a), \quad I(0) = I_0, \quad \text{and} \quad R(0) = 0,$$

Simulations: plateaus and rebounds





Model calibration on Thau lagoon data (Sète)



Conclusions

- “epidemiology, […] variation of disease from time to time or from place to place…”
- Models with diffusion allow one to consider complex phenomena in epidemics
- Diffusion comes in various guises in epidemiology
- Spatial propagation
- Propagation on networks
- Modeling the effect of roads
- Taking into account behavioral heterogeneity and variability
- Generate complex dynamics with plateaus and rebounds
- Open new mathematical challenges
- New modelling developments in epidemiology and social sciences